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Reg. No. : ....

Name : ....

Fourth Semester B.Tech. Degree Examination, May 2013
(2008 Scheme)
08.401 : ENGINEERING MATHEMATICS – III (CMPUNERFHB)

Time : 3 Hours Max. Marks : 100

Instructions : *Answer **all** questions from Part – **A** and **one full** question from **each** Module of Part – **B**.*

JOHNCOX MEMORIAL CSI INSTITUTE OF TECHNOLOGY
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PART – A

1. Prove that the function e^{y+ix} is nowhere differentiable.

2. If u and v are harmonic functions prove that $\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)+i\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$ is analytic.

3. Prove that an analytic function with constant imaginary part is a constant.

4. Find the image of $|z-3i|=3$ under $w=\frac{1}{z}$.

5. By Cauchy's integral formula evaluate $\int_C \frac{\sin^2 z}{\left(z-\frac{\pi}{2}\right)^3} . dz$ where c is $|z| = 2$.

6. Find the Taylor's series expansion of $\frac{1}{z^2}$ about $z = 2$. State the region of validity.

7. Find the poles and residues of $\frac{1+e^z}{\sin z + z\cos z}$.

P.T.O.

8. Find a positive root of $xe^x = 2$ by the method of false positions by performing 4 iterations.

9. Find a root of $x^3 - 2x - 5 = 0$ using Newton-Raphson method.

10. Evaluate $\int_0^6 \frac{dx}{1+x}$ using Trapezoidal rule by dividing the interval into 6 equal parts.

(10×4=40 Marks)

PART – B

Module – 1

11. a) Show that $f(z) = \begin{cases} \frac{xy^2}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ satisfy CR equations, but not differentiable at z = 0.

b) Find the analytic function f (z) = u + iv if $u - v = e^x (\cos y - \sin y)$.

c) Determine the region of the w-plane into which the triangular region bounded by x = 1, y = 1 and x+y = 1 is mapped by $w = z^2$.

12. a) Show that $u(x, y) = x^2 - y^2$ and $v(x, y) = \frac{-y}{x^2+y^2}$ are both harmonic, but u + iv is not analytic.

b) If the potential function is $\log(x^2 + y^2)$, find the flux function and the complex potential function.

c) Find the bilinear transformation which maps the points (2, i, –2) into the points (1, i, –1).

Module – 2

13. a) Evaluate $\int_C |z| \bar{z}\, dz$ where C consists of the upper semicircle $|z| = 1$ and the segment $-1 \le x \le 1$.

b) Find the Laurent's series expansion of $\frac{z^2-1}{(z+2)(z+3)}$ in $|z| > 3$.

c) $I = \int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, evaluate I using Cauchy's Residue theorem where C is $|z| = 3$.

14. a) Evaluate $\int_0^{\pi} \frac{d\theta}{(2+\cos\theta)^2}$

b) Evaluate $\int_0^{\infty} \frac{dx}{1+x^6}$

Module – 3

15. a) Solve the following system of equations by Gauss Seidal iteration method.

$28x + 4y - z = 32$, $x + 3y + 10z = 24$, $2x + 17y + 4z = 35$.

b) The population of a town is as follows :

Year	**x**	1941	1951	1961	1971	1981	1991
Population in lakhs	**y**	20	24	29	36	46	51

Estimate the population increase during the period 1946 to 1976 by Newton's formula.

c) Using Lagrange's formula, fit a polynomial for the following data.

x :	1	2	7	8
y :	4	5	5	4

Find the value of y when x = 6.

16. a) The speeds of a train at various times are given below :

t (hr)	0	.5	1	1.5	2	2.5	3	3.5	4
v (in kmph)	0	13	33	39.5	40	40	36	15	0

Find the total distance covered using Simpson's method.

b) Solve by Euler's method $\frac{dy}{dx} = x + y^2$ and y (0) = 0. Find y approximately for x = .5 by taking h = 0.1.

c) Apply Runge-Kutta method of fourth order to find an approximate vlaue of y when x = 0.7 , given that $\frac{dy}{dx} = y - x^2$ and y (.6) = 1.7379. **(20×3=60 Marks)**

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